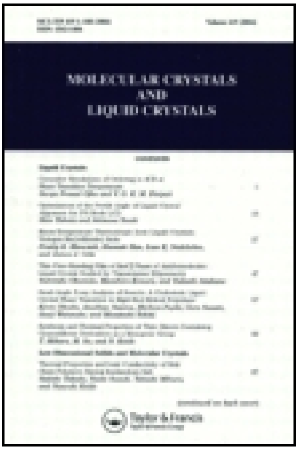

Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl19

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Version of record first published: 24 Sep 2006

Mol. Cryst. and Liq. Cryst., 2001, Vol. 359, pp.203-216
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Fluctuations and Light Scattering in Nematic Liquid Crystals in the Presence of Flexoelectric Effect

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The behavior of nematic liquid crystal with homeotropic orientation in an external electric field is considered when the flexoelectric effect exists. At the critical field E_c there is phase transition from homeotropic structure to periodic flexoelectric pattern. The dependence of the threshold parameters on the anchoring energy is analyzed in detail. The correlation functions of director fluctuations are calculated. It is shown that these function have a pole in the vicinity of the transition point at the wave number coinciding with one for the periodic structure. The angular dependence of light scattering intensity is studied when the field close to the critical value.

Keywords: Nematic; Frederiks transition; flexoelectric effect; fluctuations

1 INTRODUCTION

Nematic liquid crystals (NLC) undergo various structural transformations being placed in an external electric field. The most known effect is the Frederiks transition caused by the anisotropy of the permittivity. The structure to be appeared in this transition may be either uniform or periodic [1]–[6]. There are nematics exhibiting a specific flexoelectric effect connected with the anisotropy of molecules possessing permanent dipole moments [7]. The behavior of such systems was investigated in Refs.[2, 8] for the planar geometry and under assumption of strong anchoring with bounded surface. The influence of flexoelectric polarization on the orientational phase transition was analyzed.

Recently, considerable attention is aimed to the study of nematic interaction with bounded surface [5, 6, 10]. For this reason the Frederiks transition is interest due to its sensitive to the NLC parameters. Therefore this effect gives the possibility to determine real characteristics of nematics and, in particular, the surface energy. For planar geometry this problem is cumbersome and can be solved numerically only. We consider the cell of NLC with homeotropic orientation. In this case we were able to obtain very simple analytical expression determining the threshold parameters in dependence on the anchoring energy, the Frank modules and the electric constants of the nematic. It is shown that in contrast to the planar geometry the periodic structure is realized here due to flexoelectric effect only. We obtain the expressions for the correlation functions of director fluctuations which have poles near the threshold. The intensity of scattered light in bounded nematic was calculated. The angular dependence of the intensity has a sharp peak when scattering vector corresponds to the period of the pattern.

The paper is organized as follows. In Section II the general equations are presented and the periodic flexoelectric structure is considered, Section III is devoted to the study of light scattering near the threshold.

2 STRUCTURE NLC IN AN EXTERNAL FIELD

Let us consider a layer of nematic liquid crystal of thickness d with initial uniform homeotropic orientation of the director $\mathbf{n^0}$. We introduce the Cartesian coordinate frame with the origin at the cell centre and z axis directed along $\mathbf{n^0}$, normal to the layer. In the presence of an external electric field $\mathbf{E} = (E, 0, 0)$ the distribution of the director changes, $\mathbf{n} = (n_x(y,z), n_y(y,z), 1)$. The variation of the free energy, up to the terms of the second order in n_x, n_y, has the form

$$F = \frac{1}{2}\int d\mathbf{r}\, \{K_1(\partial_y n_y)^2 + K_2(\partial_y n_x)^2 + K_3[(\partial_z n_x)^2 + (\partial_z n_y)^2] - \epsilon E^2 n_x^2 - 2eEn_x\partial_y n_y\}, \quad (2.1)$$

where K_i (i = 1,2,3) are the Frank modules, $e = e_1 - e_3$ is the difference of the flexoelectric coefficients, $\epsilon = \varepsilon_a/4\pi$, $\varepsilon_a = \varepsilon_{\|} - \varepsilon_{\perp}$ is the permittivity anisotropy, $\varepsilon_{\parallel}, \varepsilon_{\perp}$ are the dielectric permittivities along and across to the nematic axis respectively (for definiteness we consider $\varepsilon_a > 0$), the symbol ∂_j ($j = x, y, z$) means the partial derivative on an appropriate coordinate.

The equilibrium distribution providing the extremum of the functional F satisfies to the Euler equations

$$\begin{cases} K_2\partial_y^2 n_x + K_3\partial_z^2 n_x + eE\partial_y n_y + \epsilon E^2 n_x = 0 \\ K_1\partial_y^2 n_y + K_3\partial_z^2 n_y - eE\partial_y n_x = 0. \end{cases} \tag{2.2}$$

Taking into account the symmetry of the system (2.2) we will seek the solution in the form

$$n_x = \theta_1(z)\sin(py), \quad n_y = \theta_2(z)\cos(py). \tag{2.3}$$

Inserting Eq.(2.3) into Eq.(2.2) we have

$$\widehat{A}\begin{pmatrix}\theta_1(z)\\ \theta_2(z)\end{pmatrix} = 0\,, \tag{2.4}$$

where

$$\widehat{A} = \begin{pmatrix} -K_2p^2 + \epsilon E^2 + K_3\partial_z^2 & -eEp \\ -eEp & -K_1p^2 + K_3\partial_z^2 \end{pmatrix}.$$

To solve Eq.(2.4) we diagonalize matrix $\widehat{A}$ by the transformation $\widehat{U}^{-1}\widehat{A}\widehat{U}$, where $\widehat{U}$ is the matrix of the eigenvectors

$$\begin{aligned} \widehat{U} &= \begin{pmatrix} 2eEp & 2eEp \\ f - g & f + g \end{pmatrix}, \\ f = f(p, E) &= (K_1 - K_2)p^2 + \epsilon E^2, \\ g = g(p, E) &= \sqrt{f^2 + (2eEp)^2}. \end{aligned} \tag{2.5}$$

Multiplying Eq.(2.4) from the left by $\widehat{U}^{-1}$ and introducing new variables

$$\begin{pmatrix}\theta_1'(z)\\ \theta_2'(z)\end{pmatrix} = \widehat{U}^{-1}\begin{pmatrix}\theta_1(z)\\ \theta_2(z)\end{pmatrix}, \tag{2.6}$$

we obtain

$$\begin{pmatrix} -Q^2 + \partial_z^2 & 0 \\ 0 & -P^2 + \partial_z^2 \end{pmatrix}\begin{pmatrix}\theta_1'(z)\\ \theta_2'(z)\end{pmatrix} = 0. \tag{2.7}$$

Here

$$P = P(p, E) = \sqrt{[(K_1 + K_2)p^2 - \epsilon E^2 + g]/2K_3}, \tag{2.8}$$

$$Q = Q(p, E) = \sqrt{[(K_1 + K_2)p^2 - \epsilon E^2 - g]/2K_3}. \tag{2.9}$$

The solution of the system (2.7) is of the form

$$\begin{pmatrix} \theta_1'(z) \\ \theta_2'(z) \end{pmatrix} = \begin{pmatrix} a_1 e^{Qz} + a_2 e^{-Qz} \\ b_1 e^{Pz} + b_2 e^{-Pz} \end{pmatrix}, \tag{2.10}$$

where a_i, b_i ($i = 1,2$) are integration constants. Using Eqs.(2.3),(2.6),(2.6), and (2.10) we get the expressions for the director components $n_x(y, z)$ and $n_y(y, z)$

$$n_x = 2eEp\sin(py)\sum_{i=1}^{2}\left\{a_i\exp[(-1)^{i+1}Qz] + b_i\exp[(-1)^{i+1}Pz]\right\},$$

$$n_y = \cos(py)\sum_{i=1}^{2}\left\{(f+g)a_i\exp[(-1)^{i+1}Qz] + (f-g)b_i\exp[(-1)^{i+1}Pz]\right\}. \tag{2.11}$$

The coefficients a_i, b_i can be determined from the boundary conditions [1, 2]

$$\begin{cases} W\theta_1 \pm K_3\partial_z\theta_1 = 0, & z = \pm d/2 \\ W\theta_2 \pm K_3\partial_z\theta_2 = 0, & z = \pm d/2, \end{cases} \tag{2.12}$$

where W is the anchoring energy. Inserting Eq.(2.11) into Eq.(2.12) and setting the determinant of the obtained system equal to zero, we find the condition for the existence of a nontrivial solution

$$\det\begin{pmatrix} \hat{S}_Q & \hat{S}_P \\ (f+g)\hat{S}_Q & (f-g)\hat{S}_P \end{pmatrix} = 0,$$

where

$$\hat{S}_Q = \begin{pmatrix} (W + K_3Q)\exp(Qd/2) & (W - K_3Q)\exp(-Qd/2) \\ (W - K_3Q)\exp(-Qd/2) & (W + K_3Q)\exp(Qd/2) \end{pmatrix},$$

$$\hat{S}_P = \begin{pmatrix} (W + K_3P)\exp(Pd/2) & (W - K_3P)\exp(-Pd/2) \\ (W - K_3P)\exp(-Pd/2) & (W + K_3P)\exp(Pd/2) \end{pmatrix}.$$

It is easy to verify that this condition can be reduced to the following equation

$$\left(\tanh(Qd) + \frac{2\xi Q}{1+\xi^2 Q^2}\right)\left(\tanh(Pd) + \frac{2\xi P}{1+\xi^2 P^2}\right) = 0, \tag{2.13}$$

where $\xi = K_3/W$ is the characteristic length determining an influence of the limiting surface on the NLC orientation.

As can be seen from Eqs.(2.6),(2.8) the value P is real for any values of E and p. Then, taking into account Eq.(2.13), we get

$$\text{tg}\ \rho = \frac{2h\rho}{\rho^2 - h^2}, \tag{2.14}$$

where $h = d/\xi$, $\rho = iQd$. By virtue of the function $|Q(E)|$ grows monotonously, the smallest root of Eq.(2.14) determines E dependence on p.

$$E(p) = \sqrt{\frac{(\rho^2 d^{-2} K_3 + K_2 p^2)(\rho^2 d^{-2} K_3 + K_1 p^2)}{\rho^2 d^{-2} \epsilon K_3 + (\epsilon K_1 + e^2)\, p^2}}.$$

Minimizing this function we obtain the critical values for the wave number and the field

$$p_c = \frac{\rho}{d}\sqrt{\frac{\mu K_3}{e^2 + \epsilon K_1}}, \quad E_c = \frac{\rho}{d}\sqrt{\frac{K_3}{\epsilon}}\ \delta, \tag{2.15}$$

where

$$\mu = e\sqrt{\frac{e^2 + \epsilon(K_1 - K_2)}{K_1 K_2}} - \epsilon, \tag{2.16}$$

$$\delta = \sqrt{\frac{\epsilon}{\mu + \epsilon}\left(1 + \frac{\mu K_2}{e^2 + \epsilon K_1}\right)\left(1 + \frac{\mu K_1}{e^2 + \epsilon K_1}\right)}.$$

Let us consider the limiting cases.

For the strong anchoring of the director with the bounded surfaces, $W = \infty$, the boundary conditions (2.12) take the form

$$\theta_1(\pm d/2) = 0, \quad \theta_2(\pm d/2) = 0. \tag{2.17}$$

From Eq.(2.14) we get $\rho = \pi$. In one-constant approximation, $K_1 = K_2 = K_3$, Eqs.(2.15) coincide with the results of Ref.[9]. The asymptotics of the critical field at $W \to \infty$ is

$$E_c \approx E_c^\infty\left(1 - \frac{2K_3}{Wd}\right)$$

where $E_c^\infty = \pi\delta d^{-1}\sqrt{K_3/\epsilon}$ is the threshold value for strong anchoring [1, 2].

For unlimited sample, where $W = 0$, from Eq.(2.14) we get $\rho = 0$, and taking into account Eq.(2.9) we obtain the linear dependence $E(p)$

$$E = p\sqrt{\frac{K_1K_2}{\epsilon K_1 + e^2}}.$$

Thus, the effect has no threshold, i.e.$E_c^0 = 0$, and each value of field corresponds the definite meaning of the flexoelectric structure period [2]. At small energy W the asymptotic behavior of the critical value is

$$E_c \approx \sqrt{\frac{2W}{\epsilon d}}\,\delta\,.$$

It is obvious that ρ grows monotonously with the increase of W from $\rho(W = 0) = 0$ to $\rho(W = \infty) = \pi$.

From Eqs.(2.15), (2.16) follows that the type of the structure arising in homeotropically oriented cell in the presence of an external electric field depends on the sign of $\mu(e, \epsilon, K_1, K_2)$.

At $\mu > 0$, i.e. $\epsilon K_2/e^2 < 1$, the function $E(p)$ has a minimum in the point $p_c \neq 0$. In this case there is the phase transition from uniform homeotropic system to a periodic flexoelectric structure with the wave number p_c. Appearance of the pattern is governed by the Frank module K_2 connected with twist deformation arising in this structural transformation. The existence of the flexoelectric polarization, $e \neq 0$, is the necessary condition for the transition. It is the basic difference from a planar cell where there is a possibility to form the periodic pattern in the absence of the flexoelectric effect [2]–[8].

At $\mu \leqslant 0$, i.e. $\epsilon K_2/e^2 \geqslant 1$, the usual Frederiks effect takes place.

The obtained criterion does not depend on the value of anchoring energy.

Fig.1 shows the dependence of the critical wave number of the periodic flexoelectric structure on the anchoring energy W. The dependence of the threshold field strength U_c on the cell thickness d for various W is presented in Fig.2.

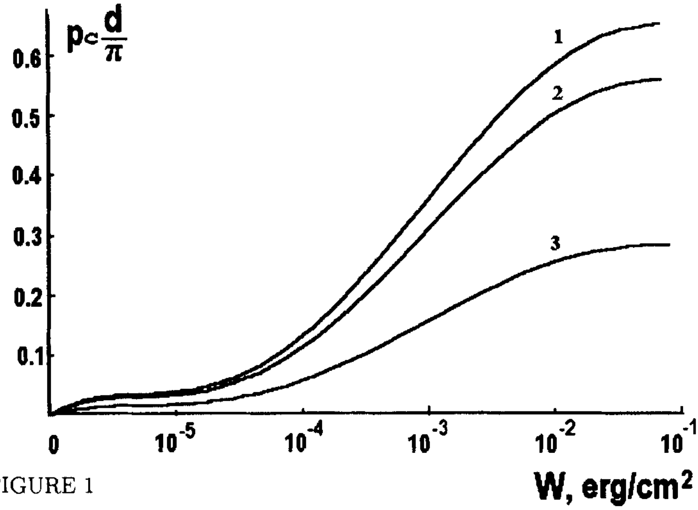

FIGURE 1

The dependence of the critical wave number q_c on the anchoring energy W for different values of the flexoelectric coefficients e: 1 — $0.33 \cdot 10^{-11}$ C/m, 2 — $0.5 \cdot 10^{-11}$ C/m, 3 — $0.67 \cdot 10^{-11}$C/m. The other NLC parameters used are: $\varepsilon_a = 0.1$, $K_1 = 0.7 \cdot 10^{-6}$ dyn, $K_2 = 10^{-6}$ dyn, $_3 = 0.5 \cdot 10^{-6}$ dyn, $d = 10^{-3}$cm

3 LIGHT SCATTERING BY DIRECTOR FLUCTUATION IN TRANSITION REGION

The considered threshold effect refers to the second order phase transitions characterized by fluctuations growth near the critical point. The most effective method for studying of the director fluctuations is the light scattering. To describe this phenomenon for the NLC layer it is convenient to expand the director fluctuations $\delta\mathbf{n}(\mathbf{r})$ in the two-dimensional Fourier spectrum.

$$\delta\mathbf{n}(\mathbf{r}) = \frac{1}{(2\pi)^2} \int d\mathbf{q} \, \exp[-i(q_x x + q_y y)] \, \delta\mathbf{n}(\mathbf{q}, z),$$

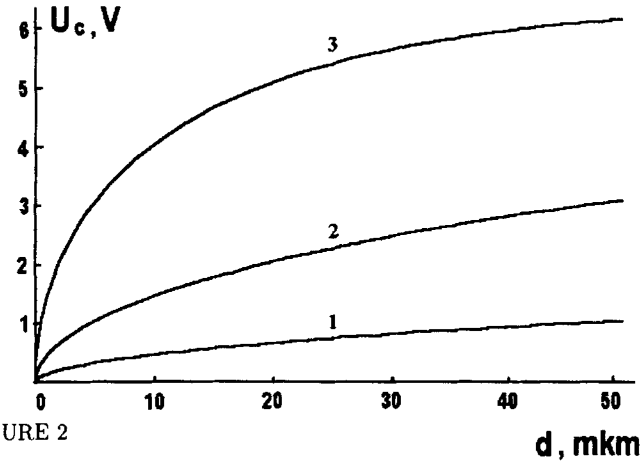

FIGURE 2

The dependence of the threshold electric field strength on the cell thickness d for different values of W and $e = 0.33 \cdot 10^{-11}$ C/m: 1 — 0.001 erg/cm^2, 2 — 0.01 erg/cm^2, 3 — 0.1 erg/cm^2.

where $\mathbf{q} = (q_x, q_y)$ is the wave vector of the fluctuation mode.
In our geometry the arising structure is uniform along x axis, therefore the critical phenomena in nematic appear in the modes with $q_x = 0$ [8, 9], and in what follows we consider the wave vectors $\mathbf{q} = (0, q, 0)$ only.

For simplicity we restrict our consideration to the case of strong anchoring, i.e. $\delta\mathbf{n}(q, z = \pm d/2) = 0$. The contribution to the free energy associated with director fluctuations is the Frank energy (2.1) which can be presented in the form

$$F = \frac{1}{2\pi} \int dq F_q,$$

where

$$F_q = \frac{1}{2} \int\limits_{-d/2}^{d/2} dz\ \delta\mathbf{n}^*(q, z)\ \widehat{B}\ \delta\mathbf{n}^t(q, z),$$

$$\widehat{B} = \begin{pmatrix} K_2q^2 - \epsilon E^2 - K_3\partial_z^2 & ieEq \\ -ieEq & K_1q^2 - K_3\partial_z^2 \end{pmatrix}$$

The superscript ^{t} means transposition and * shows complex conjugation.

To determine the scattered light intensity it is necessary to know the correlation functions of the director fluctuations $\widehat{G}(q,z,z') = \langle \delta\mathbf{n}(q,z)\delta\mathbf{n}^t(q,z')\rangle$, which satisfies the equation [9, 10]

$$\widehat{B}\widehat{G}(q,z,z') = k_BT\hat{I}\delta(z-z'), \tag{3.1}$$

where T is the temperature, k_B is the Boltzmann constant, $\hat{I}$ is the unit matrix, $\delta(z-z')$ is the delta-function.

In order to calculate the correlation functions one must convert the operator $\widehat{B}$ taking into account the boundary conditions

$$\widehat{G}\left(q, z = \pm\frac{d}{2}, z'\right) = 0. \tag{3.2}$$

Matrix $\widehat{B}$ is diagonalized by the transformation $\widehat{V}^{-1}\widehat{B}\widehat{V}$, where

$$\widehat{V} = \begin{pmatrix} 2ieEq & 2ieEq \\ f+g & f-g \end{pmatrix}.$$

The functions $f = f(q,E)$, $g = g(q,E)$ are determined by Eq.(2.6).

Eq.(3.1) may be written in the equivalent form

$$\widehat{V}^{-1}\widehat{B}\widehat{V}\widehat{V}^{-1}\widehat{G}\widehat{V} = k_BT\hat{I}\delta(z-z'),$$

Using the explicit forms of $\widehat{B}$, $\widehat{U}$ and $\widehat{U}^{-1}$ matrices we have

$$\begin{pmatrix} -P^2 + \partial_z^2 & 0 \\ 0 & -Q^2 + \partial_z^2 \end{pmatrix} \widehat{X} = -\frac{k_BT}{K_3}\hat{I}\delta(z-z'), \tag{3.3}$$

where

$$\widehat{X} = \widehat{V}^{-1}\widehat{G}\widehat{V}, \tag{3.4}$$

and $P = P(q,E)$, $Q = Q(q,E)$ are the functions introduced in Eqs.(2.8),(2.9). The boundary conditions (3.2) require that

$$\widehat{X}\left(z = \pm\frac{d}{2}, z'\right) = 0.$$

Thus, we get the system of four differential equations for four elements of $\widehat{X}$ matrix. It is evidently that

$$X_{12}(z,z') = X_{21}(z,z') = 0. \tag{3.5}$$

The diagonal elements $X_{11}(z,z')$ and $X_{22}(z,z')$ one can find from the problem of the type

$$\begin{cases} \partial_z^2 X_{ii}(z,z') - \alpha_i^2 X_{ii}(z,z') = \beta_i \delta(z-z') \\ X_{ii}(z = \pm\frac{d}{2}, z') = 0 \end{cases}, \quad i = 1,2.$$

Its solution is

$$X_{ii}(z,z') = \begin{cases} X_{ii}^{(+)} = C^{(+)} \sinh\left[\alpha_i \left(z - \frac{d}{2}\right)\right], & z > z' \\ X_{ii}^{(-)} = C^{(-)} \sinh\left[\alpha_i \left(z + \frac{d}{2}\right)\right], & z < z', \end{cases} \tag{3.6}$$

where $C^{(++}, C^{(-)}$ are constants determined by the conditions imposed on functions $X_{ii}^{(+)}$ and $X_{ii}^{(-)}$ in the point $z = z'$

$$\begin{cases} X_{ii}^{(+)}(z=z') = X_{ii}^{(-)}(z=z') \\ \partial_z X_{ii}^{(+)}(z=z') - \partial_z X_{ii}^{(-)}(z=z') = \beta_i, \end{cases} \tag{3.7}$$

If we solve the system (3.7)and insert the obtained coefficients $C^{(+)}, C^{(-)}$ into Eq.(3.6), we get

$$X_{ii}(z,z') = \frac{\beta_i}{\alpha_i \sinh(\alpha_i d)} \begin{cases} \sinh\left[\alpha_i\left(z-\frac{d}{2}\right)\right] \sinh\left[\alpha_i\left(z'+\frac{d}{2}\right)\right], & z > z' \\ \sinh\left[\alpha_i\left(z+\frac{d}{2}\right)\right] \sinh\left[\alpha_i\left(z'-\frac{d}{2}\right)\right], & z < z', \end{cases}$$

Or, in more compact form

$$X_{ii}(z,z') = \frac{\beta_i}{2\alpha_i \sinh(\alpha_i d)} \Big\{ \cosh\left[\alpha_i (z+z')\right] \\ - \cosh(\alpha_i d) \cosh\left[\alpha_i (z-z')\right] + \sinh(\alpha_i d) \sinh(\alpha_i |z-z'|) \Big\}. \tag{3.8}$$

Using the explicit forms for coefficients α_i and β_i, Eq.(3.3), we have

$$X_{11}(z,z') = \frac{k_B T}{2K_3 P \sinh(Pd)} \Big\{ -\cosh[P(z+z')] + \cosh(Pd)\cosh[P(z-z')] \\ - \sinh(Pd)\sinh(P|z-z'|) \Big\} \equiv -\frac{1}{K_3}\eta(P), \tag{3.9}$$

$$X_{22}(z,z') = \frac{k_B T}{2K_3 Q \sinh(Qd)} \Big\{ -\cosh[Q(z+z')] + \cosh(Qd)\cosh[Q(z-z')] - \sinh(Qd)\sinh(Q \mid z-z' \mid) \Big\} \equiv -\frac{1}{K_3}\eta(Q). \quad (3.10)$$

Taking into account Eqs.(3.4), (3.5), (3.9), and (3.10) we get the correlation matrix

$$G_{11} = \langle n_x(q,z) n_x^*(q,z') \rangle = \frac{1}{2K_3}\left[\left(\frac{f}{g}-1\right)\eta(P) - \left(\frac{f}{g}+1\right)\eta(Q)\right],$$

$$G_{22} = \langle n_y(q,z) n_y^*(q,z') \rangle = -\frac{1}{2K_3}\left[\left(\frac{f}{g}+1\right)\eta(P) + \left(\frac{f}{g}-1\right)\eta(Q)\right],$$

$$G_{12} = \langle n_x(q,z) n_y^*(q,z') \rangle = -\frac{ieEq}{gK_3}\left[\eta(P) - \eta(Q)\right],$$

$$G_{21} = \langle n_y(q,z) n_x^*(q,z') \rangle = \frac{ieEq}{gK_3}\left[\eta(P) - \eta(Q)\right].$$

At $E = 0$ these expressions coincide with known results for strong anchoring. Note, the correlation functions have a pole of the first order for $q = p_c$ when the electric field approaches the threshold value E_c.

The tensor of permittivity $\varepsilon_{\alpha\beta}$ is connected with the director by the relation [12]

$$\varepsilon_{\alpha\beta}(\mathbf{r}) = \varepsilon_\perp \delta_{\alpha\beta} + \varepsilon_a n_\alpha(\mathbf{r}) n_\beta(\mathbf{r}).$$

Fluctuations of the director $\delta\mathbf{n}(\mathbf{r})$ determine the variation of this tensor

$$\delta\varepsilon_{\alpha\beta}(\mathbf{r}) = \varepsilon_a[n_\alpha \delta n_\beta(\mathbf{r}) + n_\beta \delta n_\alpha(\mathbf{r})]. \quad (3.11)$$

This change, in turn, causes the light scattering. Its intensity I is proportional to $\langle E'_\alpha(\mathbf{r}) E'^*_\beta(\mathbf{r}) \rangle$, where $\mathbf{E}'$ is the field of the scattered wave [12, 13]. If the incident field is a plane wave $\mathbf{E}^0 e^{i\mathbf{k}_i\mathbf{r}}$ with the wave vector $\mathbf{k}_i$ the value $\langle E'_\alpha(\mathbf{r}) E'^*_\beta(\mathbf{r}) \rangle$ is equal to [11, 13]

$$\langle E'_\alpha(\mathbf{r}) E'^*_\beta(\mathbf{r}) \rangle = \frac{\omega^4}{c^4} \int d\mathbf{r}' d\mathbf{r}'' T_{\alpha\gamma}(\mathbf{r},\mathbf{r}') T^*_{\beta\lambda}(\mathbf{r},\mathbf{r}'') \times \langle \delta\varepsilon_{\gamma\mu}(\mathbf{r}') \delta\varepsilon_{\lambda\nu}(\mathbf{r}'') \rangle E^0_\mu E^0_\nu \exp[i\mathbf{k}_i(\mathbf{r}' - \mathbf{r}'')], \quad (3.12)$$

where ω is the circular frequency, c is the velocity of light, $T_{\alpha\beta}(\mathbf{r}',\mathbf{r}'')$ is the Green function of the Maxwell equations; integration is carried out over the scattered volume.

For simplicity we use the approximation of an isotropic medium. In this case the Green function in the far zone is equal to

$$T_{\alpha\beta}(\mathbf{r}) = \frac{1}{4\pi r} e^{i\mathbf{kr}} (\delta_{\alpha\beta} - s_\alpha s_\beta),$$

where $k = \omega\sqrt{\varepsilon}/c$, ε is average permittivity, $\mathbf{s} = \mathbf{r}/r$ is the direction to the observation point. Then

$$\langle E'_\alpha(\mathbf{r}) E'^*_\beta(\mathbf{r})\rangle = \frac{\omega^4 V}{c^4 (4\pi)^2 r^2} (\delta_{\alpha\gamma} - s_\alpha s_\gamma)(\delta_{\beta\lambda} - s_\beta s_\lambda) \frac{1}{d} \int_{-d/2}^{d/2} dz' \int_{-d/2}^{d/2} dz'' \times e^{-iq_{sc,z}(z - z')} \langle \delta\varepsilon_{\gamma\mu}(\mathbf{q}_{sc}^\perp, z') \delta\varepsilon^*_{\lambda\nu}(\mathbf{q}_{sc}^\perp, z'')\rangle E^0_\mu E^0_\nu, \quad (3.13)$$

where V is scattering volume, $\mathbf{q}_{sc} = k\mathbf{s} - \mathbf{k}_i$ is the vector of scattering, $\mathbf{q}_{sc}^\perp = (q_{sc,x}, q_{sc,y}, 0)$ is the transversal component of $\mathbf{q}_{sc}$. In our case $q_{sc,x} = 0$, $q_{sc,y} = q$. According to Eq.(3.11) the correlation function of permittivity fluctuations is connected with the director fluctuations by the relation

$$\begin{aligned} &\langle \delta\varepsilon_{\gamma\mu}(q, z') \delta\varepsilon^*_{\lambda\nu}(q, z'')\rangle \\ &= \varepsilon_a \Big[n^0_\gamma n^0_\lambda \langle \delta n_\mu(q, z') \delta n_\lambda(q, z'')\rangle + n^0_\mu n^0_\nu \langle \delta n_\gamma(q, z') \delta n_\lambda(q, z'')\rangle \\ &+ n^0_\gamma n^0_\nu \langle \delta n_\mu(q, z') \delta n_\lambda(q, z'') > + n^0_\mu n^0_\lambda < \delta n_\gamma(q, z') \delta n_\nu(q, z'')\rangle \Big]. \quad (3.14) \end{aligned}$$

Let us consider the case of normal incidence, $\mathbf{k}_i = (0, 0, k)$. For scattering in $\mathbf{s} = (0, \sin\theta, \cos\theta)$ direction vector $\mathbf{q}_{sc}$ is

$$\mathbf{q}_{sc} = k(0, \sin\theta, \cos\theta - 1), \; q_{sc} = 2k\sin(\theta/2).$$

If polarization of the incident light is directed along y axis, i. e. $\mu = \nu = y$, the intensity of scattered light is equal to

$$I(\theta) = I_0 \frac{\omega^4}{c^4} \frac{V\epsilon^2}{r^2 d} \sin^2\theta \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} dz' dz'' e^{-iq_{sc,z}(z' - z'')} G_{22}(q, z, z')$$

Calculating integrals we finally get

$$I = \frac{I_0}{2} \sin^2\theta \frac{\epsilon^2 C}{dK_3} F(P, Q),$$

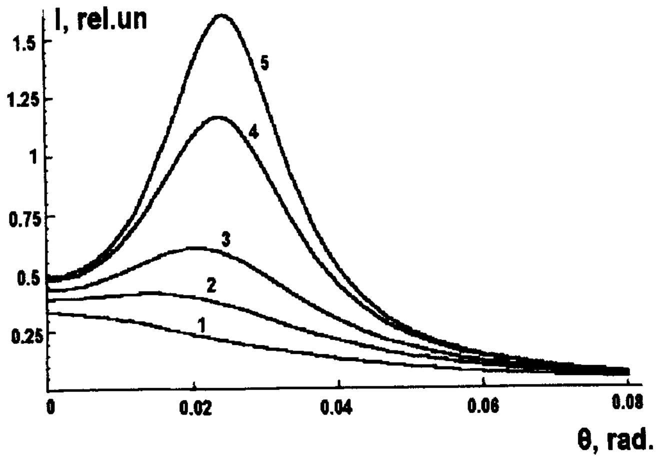

FIGURE 3

The angular dependence of the scattered light intensity for different values of the electric field strength: 1 – 0, 2 – 4.5 V, 3 – 5.4 V, 4 – 6.03 V, 5 – 6.12 V. The other NLC parameters are the same as on Fig.1

$$F(P,Q) = -\left(1+\frac{f}{g}\right)\left[\frac{2P(\cosh(Pd)-\cos(q_{sc,z}d))}{(P^2+q_{sc,z}^2)^2\sinh(Pd)} - \frac{d}{P^2+q_{sc,z}^2}\right]$$

$$-\left(1-\frac{f}{g}\right)\left[\frac{2Q(\cosh(Qd)-\cos(q_{sc,z}d))}{(Q^2+q_{sc,z}^2)^2\sinh(Qd)} - \frac{d}{Q^2+q_{sc,z}^2}\right], \qquad (3.15)$$

$$C = k_B T\frac{\omega^4}{c^4}\frac{V}{r^2}.$$

The angular dependence of scattered light intensity for various values of external field is presented in Fig.3. It is seen that the intensity has a peak in the vicinity $q = p_c$, which grows at $E \to E_c$. The existence of the sharp peak gives a good opportunity for check of the theoretical description and for experimental determination the difference of flexoelectric coefficients ϵ.

Acknowledgments

This work was partly supported by the International Soros Science Education Program and by the Russian Fund for Basic Research through Grant No. 98-02-18201.

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